A Computational Model of Accretion Disks: Visualizing Physical Dependencies with Adjustable Parameters

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Abstract:

Accretion disks in the field of astrophysics are a commonality across a variety of celestial objects. The composition and formation of accretion disks revolve around the physics of fast-moving, dense, and compact debris of neutron stars and black holes. This project presents a visualization for the accretion disk of a Shakura-Sunyaev model using Python libraries and dependencies. The program utilizes user-based input of the mass accretion rate $(\dot{M})$ *and a viscosity parameter* (α)*. The program utilizes Newtonian physics and a pseudo-relativistic inner radius to determine the ISCO (innermost stable circular orbit). Temperature distribution across the model is determined using the Stefan-Boltzmann law with a gradient graph visualizing changing temperatures based on user input values.*

The program proves that an increase in $\dot{M}$ *raises the disk temperature specifically towards the inner radius of the model, resulting in a brighter and more luminous inner radius. The viscosity* (α) *has a lesser effect on the temperature of the model, however in addendum with a higher accretion rate the material and chemical contents of the accretion disk can vary results with regards to the color spectrum, heat, and state of matter of the accretion disk. Orbital speed and density gradients emerge naturally from the Keplerian velocity distribution. This simulation demonstrates how key physical parameters influence observable properties of accretion disks and provides an educational tool for exploring these astrophysical phenomena.*

Introduction:

Accretion disks are rotating structures of gas and dust that form around massive central objects such as black holes, neutron stars, and young stars. These disks are not only critical to the growth of compact objects but also to the emission of vast amounts of energy, particularly in the X-ray and optical spectra. Understanding the physical processes within these disks — such as viscous dissipation, angular momentum transfer, and thermal radiation — is essential for interpreting astronomical observations and modeling high-energy astrophysical phenomena.

Traditionally, accretion disks are studied using the standard thin disk model, which assumes a geometrically thin, optically thick disk in Keplerian rotation. This model, although simplified, captures the essential thermodynamic and dynamic behaviors that influence observable properties such as luminosity, temperature gradients, and spectral output. Key parameters in this model include the mass accretion rate ($\dot{M}$) and the viscosity parameter (α) both of which determine how matter flows inward and how energy is dissipated throughout the disk. This paper presents a computational simulation of a Newtonian accretion disk, implemented in Python, to visualize and analyze the effects of varying accretion rate and viscosity on disk temperature, structure, and orbital dynamics. The goal is to provide an accessible tool for exploring fundamental disk behavior and to uncover qualitative relationships between physical parameters and observable features.

Methods:

The simulation is based on the standard thin accretion disk model described by Shakura & Sunyaev (1973), assuming a steady-state, geometrically thin, optically thick disk around a central mass (such as a black hole or neutron star). The disk is assumed to be Newtonian, meaning general relativistic effects are neglected — a valid approximation for distances sufficiently far from the Schwarzschild radius.

The central equation used to determine T(r) of the disk at a given radius is:

$$T(r) = [\frac{3GM\dot{M}}{8\pi\sigma r^3}(1 - \sqrt{\frac{r_{\text{in}}}{r}})]^{1/4}$$

Where:

- G is the given gravitational constant
- M is the mass of the central object
- $\dot{M}$ is the mass accretion rate
- σ is the Stefan-Boltzmann constant
- r_{in} is the inner radius of the disk

The Keplerian Orbital Velocity at each radius is calculated using:

$$v(r) = \sqrt{\frac{GM}{r}}$$

The user uses Python to customize input values such as the central mass, the mass accretion rate, inner and outer radii and disk resolution.

Assumptions and conventions:

- The disk is assumed to be in thermal equilibrium and radiating locally as a blackbody.
- Vertical structure is ignored; the model only accounts for radial variation.
- Relativistic effects are excluded; future versions may incorporate general relativity for accuracy near the ISCO.

The code then generates plots of temperature vs. radius and velocity vs. radius, as well as simulated visualizations of the disk's appearance. These outputs allow qualitative analysis of how varying the accretion rate or mass affects disk structure and observable emission.

Results:

1. Effect of Central Mass on Disk Properties

Increasing the mass of the central object significantly affects both the temperature and orbital velocity profiles:

- **Temperature:** For a fixed accretion rate, a more massive central object increases the temperature throughout the disk, especially at inner radii. This is due to the increased gravitational potential energy being converted to thermal energy.
- **Velocity:** The orbital velocity at each radius increases with central mass, consistent with the Keplerian relation:

$$v(r) \propto \sqrt{M/r}$$

2. Effect of Accretion Rate on Temperature Profile

Varying the mass accretion rate $\dot{M}$ at constant mass affects the disk's luminosity and thermal structure:

- **Higher $\dot{M}$:** Produces a uniformly hotter disk, raising the peak effective temperature and shifting the peak emission to shorter wavelengths (UV or X-ray).
- **Lower $\dot{M}$:** results in a cooler disk releasing less intense infrared and optical bands.

3. Radial Profiles and Observational Signatures

- The temperature profile follows an approximate power law:

$$T(r) \sim r^{-3/4}$$

Flattening near the outer rims of the disk.

- The velocity profile also follows a power law:

$$v(r) \sim r^{-1/2}$$

confirming the disk in Keplerian motion.

- The combination of these profiles suggests that most of the disk's radiated energy comes from the inner regions, while the outer regions contribute more to the infrared spectrum and total disk mass.

4. Visualizations and Trends

The code's plots consistently show that:

- Peak temperature moves inward and upward with increasing M or $\dot{M}$.
- The steepest parts of the velocity curve occur closest to the inner radius.
- Changes in the inner disk affect the most observable features (e.g., X-ray brightness) in high-energy astrophysics.

Discussion:

The simulation results align with established theoretical models of accretion disks, particularly the Shakura-Sunyaev thin disk model. By adjusting the central mass and accretion rate, the program reproduces realistic temperature and velocity distributions expected around compact objects such as white dwarfs, neutron stars, or black holes.

1. Physical Implications of Mass and Accretion Rate Variations

- The observed increase in temperature and orbital velocity with central mass reinforces the role of gravity in driving disk dynamics. A more massive object deepens the gravitational potential well, thereby increasing the conversion of potential to thermal and kinetic energy.
- Similarly, increasing the accretion rate boosts energy output, demonstrating how high- $\dot{M}$ disks are more luminous and radiate at shorter wavelengths. This aligns with observations of quasars and active galactic nuclei (AGN), where high accretion rates correlate with powerful X-ray and UV emission.

These results suggest that even basic simulations can capture essential physical trends and offer insights into the conditions that lead to observable disk emissions across the electromagnetic spectrum.

2. Radial Structure and Emission Characteristics

The disk's structure—hotter and faster near the center, cooler and slower at the edges—is consistent with observations of protoplanetary disks and accretion disks in binary systems. This supports the interpretation that:

- High-energy emissions (X-rays, UV) originate from the inner disk.
- Lower-energy radiation (IR, optical) is dominated by outer regions.
  This radial variation in emission allows astronomers to infer disk structure by studying spectra from accreting systems.

3. Model Limitations and Future Extensions

While the simulation captures core features, it simplifies several key aspects:

- General Relativity: Effects such as frame dragging and spacetime curvature near black holes are neglected. Incorporating these would significantly affect predictions near the innermost stable circular orbit (ISCO).
- Disk Thickness: The model assumes a geometrically thin disk. Real disks—especially at high accretion rates—can become thick or even form tori.
- Magnetic Fields and Viscosity: The simulation abstracts out mechanisms like magneto-rotational instability (MRI), which drive angular momentum transport.

Despite these limitations, the program offers a useful, visual introduction to accretion disk physics. With added complexity, it could evolve into a teaching tool or research-grade model.

Conclusion:

This project successfully demonstrates how variations in central mass and accretion rate influence the physical properties of an accretion disk. Through computational modeling, we reproduced key behaviors predicted by classical disk theory: higher masses and accretion rates lead to increased temperatures and orbital velocities, with energy output concentrated in the inner regions of the disk. These results align with observational data from systems like active galactic nuclei, X-ray binaries, and protostars.

Although simplified, the simulation effectively illustrates the fundamental mechanisms that govern accretion physics. By adjusting a few parameters, users gain insight into how compact objects interact with their surrounding matter and how that interaction translates into the emission of radiation across the electromagnetic spectrum.

Looking forward, the model could be enhanced with relativistic effects, vertical disk structure, and magnetic field dynamics to more accurately simulate environments near black holes or neutron stars. Nonetheless, this program already serves as a valuable educational tool and a foundation for deeper explorations into the dynamic and often extreme physics of accretion disks.

References:

- **Frank, J., King, A., & Raine, D. J. (2002).** *Accretion Power in Astrophysics* (3rd ed.). Cambridge University Press.
  — A definitive textbook on the theory and physics of accretion processes.
- **Pringle, J. E. (1981).** "Accretion Discs in Astrophysics." *Annual Review of Astronomy and Astrophysics*, 19, 137–162.
  — A classic review summarizing the standard theory of accretion disks.
- **Shakura, N. I., & Sunyaev, R. A. (1973).** "Black Holes in Binary Systems: Observational Appearance." *Astronomy and Astrophysics*, 24, 337–355.
  — Introduced the influential Shakura–Sunyaev α-disk model used widely in disk theory.
- **Carroll, B. W., & Ostlie, D. A. (2017).** *An Introduction to Modern Astrophysics* (2nd ed.). Cambridge University Press.
  — A general textbook that includes accretion disk physics and radiative transfer.
- **Kato, S., Fukue, J., & Mineshige, S. (2008).** *Black-Hole Accretion Disks: Towards a New Paradigm*. Kyoto University Press.
  — Discusses advanced accretion disk models including instabilities and relativistic effects.

- **NASA HEASARC.** Accretion Disks and High-Energy Astrophysics. https://heasarc.gsfc.nasa.gov — An educational portal with data and summaries on black holes, disks, and related observations.